

NASA Technical Memorandum 87590

**The Role of Amplitude-to-Phase
Conversion in the Generation of
Oscillator Flicker Phase Noise**

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Summary

This paper contends that flicker phase noise in oscillators can be the result of a two-step process: frequency translation of low-frequency flicker noise up to the oscillation frequency by quadratic upconversion followed by an amplitude-to-phase conversion mechanism. Several such mechanisms are examined, with particular emphasis on the amplitude-to-phase conversion process associated with reactively loaded nonlinearities. It is concluded that the effect can be a significant factor in determining the phase noise density of an oscillator, but one that can be rendered small by good circuit design.

Introduction

The subject of flicker phase noise in oscillators has been studied for nearly three decades by numerous investigators (refs. 1, 2, and 3); yet, it is still an area of current interest (refs. 4 and 5). Flicker phase noise causes a broadening of the line width in an oscillator and is characterized by a phase noise power spectral density that varies inversely with offset from the carrier frequency. This behavior is also observed in the power spectrum of the envelope of the oscillator signal. It is well-known that flicker noise is present in all active devices such as semiconductor and vacuum tube diodes, bipolar junction transistors, and field effect transistors (FET's); however, the processes whereby this low-frequency noise is transferred into the phase of an oscillator signal are still not totally understood. This irresolution is no doubt due largely to the extreme difficulty associated with the mathematical analysis of nonlinear systems, and the fact that a number of different mechanisms can be involved in the generating process.

The work of Troitsky (ref. 1) is very significant, although virtually unknown in this country. He analyzed a Van der Pol negative-resistance oscillator containing a quadratic nonlinear term in addition to the usual cubic nonlinearity. He concluded that the quadratic term permitted flicker noise to be translated up to the carrier frequency as amplitude modulation (AM), and that frequency modulation (FM) occurred if the oscillation frequency did not coincide with the center frequency of the circuit. Unfortunately, the extension of this work to the more familiar feedback oscillator concept has been slow, and even now it is not widely accepted. This paper is an attempt to build on some earlier results and, hopefully, make several small contributions to the subject of flicker phase noise in oscillators.

Symbols and Abbreviations

A	magnitude of limiter input signal
A_o	value of A at equilibrium
AM	amplitude modulation
C	capacitance
C_v	variable shunt capacitance
D	diode
DSBSC	double-sideband suppressed carrier
$e_{AM}(t)$	amplitude modulated waveform
$e_{o1}(t)$	fundamental frequency component of soft-limited AM signal
FET	field effect transistor
FM	frequency modulation
f	frequency
$g()$	functional relationship for equation (1)
$h()$	functional relationship for equation (4)
m	amplitude modulation index
m'	postlimiting amplitude modulation index
n	circuit order
PM	phase modulation
R	resistance, Ω
t	time
V_b	diode switching voltage (see fig. 1)
V_{o1}	first-zone magnitude (see eq. (10))
α	modulating frequency, rad
θ	input-output phase shift of resonator
θ_o	value of $\theta(\omega)$ at ω_o
ϕ	input-output phase shift of limiter
ϕ_o	value of ϕ at A_o and ω_o
ω	oscillation frequency, rad
ω_o	operating point, rad

Flicker Noise Transfer

The main conclusion of a very recent publication (ref. 5) was that the elimination of even-order nonlinearity in a feedback oscillator would prevent frequency translation of flicker noise up to the oscillation frequency. This view has also been expressed by others (refs. 1, 2, and 3). It is the contention of this paper that this conclusion is correct, but only in a limited sense, as there are commonly encountered circuit conditions for which the conclusion is incorrect. For example, the presence of voltage-dependent capacitance in an oscillator provides a very efficient mechanism for flicker noise to directly modulate the frequency of an oscillator. Also, the conclusion is held to be incorrect when the flicker noise sources are associated with "synthesized symmetrical" limiting elements. Thus, the synthesis of odd symmetry by combining nonlinear elements that individually lack symmetry, such as with opposite polarity shunt-connected or back-to-back series-connected semiconductor diodes, is to no avail except for additive input noise.

To show why this is true, consider the elementary limiting circuit of figure 1(a) that exhibits a property necessary for the establishment of an equilibrium state in a feedback oscillator: a describing function¹ that decreases with increasing input amplitude. Assume that the diodes are identical semiconductor junctions having exponential i-v characteristics, and that any noise voltages (or currents) associated with them are much smaller than the input voltage (or the diode currents it produces). Thus, the diode states are then totally controlled by the input signal, and both the intensity of the diode noise sources and the small-signal conductance of the diodes are modulated by the input signal. The output voltage resulting from the time-varying noise sources and conductance can then be determined. Since the diode noise sources are obviously uncorrelated, there is no cancellation when they are translated up to harmonic zones of the input signal, which produces only odd harmonics because of the synthesized odd symmetry of the circuit. The combination of the nonlinearly transformed input signal and the two signal and noise interaction terms is therefore an AM or DSBSC signal in each odd zone of the output, most significantly the fundamental frequency zone.

Since one is ultimately interested in the zero crossings, or phase, of the output process, it might seem that small envelope fluctuations of the output signal are not a problem. The significance arises

when one realizes the possibility for direct amplitude-to-phase conversion in subsequent circuitry, even in the limiter itself, and realizes the enormity of flicker noise density (ref. 6).

Envelope-to-Phase Transfer Mechanisms

Assuming that it is possible for some degree of flicker noise energy to be transferred up to the carrier frequency in the form of envelope fluctuations (AM), it is a substantive theoretical question to know the extent to which AM can affect the phase of the oscillation process. As one example of such a transfer mechanism, Groszkowski (ref. 7) proved that there is a functional relationship between the harmonic content and the steady-state oscillation frequency in a single-node Van der Pol negative-resistance oscillator. His analysis was based on the principle of "reactive power balance," which states that the total reactive power associated with the circuit capacitance must be equal in magnitude to that associated with the circuit inductance. It will be stated without proof that the harmonic content in a Van der Pol oscillator, i.e., one involving only cubic nonlinearity, is an increasing function of the amplitude of oscillation. Therefore, it follows that the oscillation frequency ω is related to the magnitude A of a "nearly sinusoidal" oscillation through a relationship given by

$$\omega = g(A) \quad (1)$$

This effect would provide a transfer mechanism for converting flicker envelope fluctuations into flicker frequency fluctuations.

Another mechanism for converting flicker envelope fluctuations into frequency fluctuations is provided by the presence of nonlinear reactance in the circuit such as the voltage-dependent gate-to-source capacitance in a common-source FET. It has been shown that this effect can be significantly reduced by circuit configurations that "linearize" the nonlinear element (refs. 3 and 8). This improvement occurs only if the nonlinear capacitances are noise free.

Yet another, less widely recognized, conversion mechanism is the AM-to-PM conversion process encountered in reactively loaded nonlinearities (refs. 3 and 4). A generalized mathematical formulation is complicated and unwieldy; however, the effect is easily measured and quantified by using experimental methods. The effect was studied by Kochenburger (ref. 9) in applying describing function analysis to nonlinear control systems. Conceptually, the notion of a describing function seems quite compatible with the analysis of high-Q feedback oscillators. Kochenburger derived the magnitude and phase of the describing function for a symmetrical limiting circuit

¹ The describing function of a nonlinear two-port network is defined as the ratio of the fundamental frequency component of the output to a sinusoidal input signal.

using ideal diodes, as shown in figure 1(b). In a feedback oscillator application, the shunt capacitance in the circuit of figure 1(b) could be a consequence of the limiter driving a resonator operating "off-resonance" as a result of unavoidable time delays within the loop.

Figures 2 and 3 are adaptations of Kochenburger's results and demonstrate the dependency between the input amplitude and the magnitude and phase of the fundamental frequency component of the output. This conclusion can be reached on the basis of linear network theory, since an input signal with $A < V_b$ will be subjected to a phase shift ϕ_1 given by

$$\phi_1 = -\tan^{-1}(\omega R_1 C) \quad (2)$$

However, when $A \gg V_b$, the asymptotic value is

$$\phi_2 = -\tan^{-1} \left(\omega C \frac{R_1 R_2}{R_1 + R_2} \right) \quad (3)$$

Thus, it is reasonable to conclude that a functional relationship between the input amplitude and frequency and the output phase given by

$$\phi = h(A, \omega) \quad (4)$$

will exist when the circuit is driven by the signal $A \cos \omega t$.

When a reactively loaded limiter becomes part of an oscillator, the oscillation frequency is affected by slowly varying amplitude-induced phase changes within the loop, since the loop phase must sum to $2n\pi$. For small, slowly varying perturbations around an operating point (A_o, ω_o) , ϕ may be assumed to be essentially linear with respect to both A and ω , and equation (4) can be approximated as

$$\phi = h(A_o, \omega_o) + \frac{\partial h}{\partial A} \bigg|_{A_o} (A - A_o) + \frac{\partial h}{\partial \omega} \bigg|_{\omega_o} (\omega - \omega_o) \quad (5)$$

The partial derivative $\partial h / \partial A$ is the amplitude-to-phase slope, or conversion coefficient, and $\partial h / \partial \omega$ is the familiar phase-frequency slope. The input-output phase-frequency relationship for the linear portion of a feedback oscillator may be similarly approximated as

$$\theta(\omega) = \theta_o + \frac{\partial \theta}{\partial \omega} \bigg|_{\omega_o} (\omega - \omega_o) \quad (6)$$

in the vicinity of ω , the steady-state oscillation frequency. Equations (5) and (6) may be combined by applying the conditions that $\phi(A, \omega) + \theta(\omega)$ must sum to $2n\pi$; then the resultant expression is solved for ω , yielding:

$$\omega = \frac{2n\pi - h(A_o, \omega_o) - \frac{\partial h}{\partial A} \bigg|_{A_o} (A - A_o) + \omega_o \frac{\partial h}{\partial A} \bigg|_{\omega_o} + \omega_o \frac{\partial \theta}{\partial \omega} \bigg|_{\omega_o}}{\frac{\partial h}{\partial \omega} \bigg|_{\omega_o} + \frac{\partial \theta}{\partial \omega} \bigg|_{\omega_o}} \quad (7)$$

The sensitivity of oscillation frequency to input amplitude is therefore

$$\frac{d\omega}{dA} = \frac{-\frac{\partial h}{\partial A} \bigg|_{A_o}}{\frac{\partial h}{\partial \omega} \bigg|_{\omega_o} + \frac{\partial \theta}{\partial \omega} \bigg|_{\omega_o}} \quad (8)$$

which shows the linear dependence of frequency on amplitude, the desirability of minimizing the amplitude-to-phase conversion coefficient $\partial h / \partial A$ and maximizing $\partial \theta / \partial \omega$.

It is of significant interest to know the manner in which $\partial h / \partial A$ varies with A so that an operating point might be selected to minimize amplitude-to-phase conversion. The describing-function phase data of Kochenburger (ref. 9) were adapted in figure 4 to show the describing-function phase angle plotted against input amplitude with normalized frequency being a parameter. The solid curves in figure 4 are for values of $\omega R_1 C$ of 1 and 0.4, which produce nonlimiting ($A < V_b$), or linear, phase shifts of -45° and -22° . The largest values of $\partial h / \partial A$ occur at the onset of limiting ($A = V_b$) and are 0.118 and 0.0623 radian per unit of normalized amplitude.

It is reasonable to assume that the relationships of equations (5), (7), and (8) are applicable to dynamic situations in which the input signal amplitude A is not constant but slowly varying—even though the describing function is defined for static inputs. For the sinusoidally modulated input signal described by

$$e_{AM}(t) = A(1 + m \sin \alpha t) \cos \omega t \quad (9)$$

the corresponding output signal will be hybrid modulated, and then the fundamental frequency component can be expressed as

$$e_{o1}(t) = V_{o1}(1 + m' \sin \alpha t) \cos \left(\omega t + \phi_o + m \frac{\partial h}{\partial A} \sin \alpha t \right) \quad (10)$$

if $m \ll 1$, from equations (5) and (9). The output amplitude-modulation index m' could be obtained by a linearization procedure identical to that used to find the output phase; however, it is not required for this discussion except to note that m' will be much less than m . The preponderant sideband energy associated with $e_{o1}(t)$ will therefore be due to phase modulation.

The ratio of sideband to carrier power in a sinusoidally amplitude-modulated carrier is $m^2/2$; the corresponding ratio for a small-beta phase-modulated carrier equals one-half the squared peak phase deviation (ref. 10). Thus, the AM-to-PM conversion efficiency is $(\partial h/\partial A)^2$. The value of $\partial h/\partial A$ associated with a 45° nonlimiting phase shift was found to be 0.118 radian per unit of normalized amplitude, or -18.5 dB, and is thus a numerically significant factor. It is noteworthy that the AM-to-PM conversion process does not require even-order nonlinearity, which is obviously lacking in the odd symmetry circuits of figure 1. Therefore, narrow-band amplitude fluctuations within an oscillating loop can be transformed into phase fluctuations regardless of their origin.

A question that arose in the course of this investigation was the relevance of piecewise analysis to exponentially nonlinear elements such as semiconductor diodes. An experimental test circuit was constructed using zero-bias Schottky diodes as the limiting elements, shown in figure 1(c). The shunt capacitance C_v was adjusted to produce a nonlimiting phase shift of 45° at a test frequency of 2 MHz, and the describing-function phase angle was measured with a vector voltmeter. The resultant data were normalized to the abscissa value corresponding to the onset of limiting, or 0.24 V (root mean square), and are shown by circular symbols in figure 4. The maximum value of $\partial h/\partial A$ was estimated to be 0.181 radian per unit of normalized amplitude, or -14.8 dB, which further attests to the numerical significance of the effect, especially when one recognizes that the power spectral density of flicker noise is typically several orders of magnitude greater than that of shot and Johnson noise (ref. 6). It is therefore the conclusion of this section that flicker noise-induced angular fluctuations in a feedback oscillator can result from a two-step process: the imposition of flicker noise envelope fluctuations on the oscillation via quadratic upconversion, followed by an envelope-to-phase conversion mechanism. The ultimate angular fluctuation power density is then proportional to (1) the flicker noise densities, (2) the conversion gain associated with converting the flicker noise into amplitude modulation, and (3) the AM-to-PM power conversion coefficient $(\partial h/\partial A)^2$.

Minimization of Flicker Phase Noise

This section deduces from the results presented earlier, perhaps even to the point of speculating, various means for minimizing flicker phase noise. Any measure that can defeat either of the two steps involved in the generating process is obviously beneficial. Riddle and Trew (ref. 8) reported a 20-dB

reduction in phase noise by using a novel source-coupled FET oscillator configuration. It is this author's opinion that the major factors responsible for this improvement were (1) the reduction of direct flicker noise to FM resulting from the "back-to-back" configuration of the voltage-dependent gate-to-drain capacitances (ref. 3), and (2) the significant reduction in large-signal input and output conductance variation in a source-coupled FET pair as compared with a single FET amplifier (ref. 8), rather than any reduction in flicker noise frequency translation efficiency resulting from the odd symmetry synthesized by source coupling of the FET's. Factor (1) assumes a dominant role when the voltage-dependent capacitances become a significant fraction of the total resonating capacitance, as in microwave oscillators.

Perhaps the most obvious way to minimize AM-to-PM conversion is to minimize reactive loading of the nonlinear elements. This approach is feasible at frequencies where devices and components are more or less ideal, and not complicated by a multitude of parasitic effects ($f < 100$ MHz). At higher frequencies, the functions of amplification and limiting are normally combined in one device and the two functions are essentially inseparable. It is speculated that when such separation of functions is possible, it should be pursued. Then, the amplifier should be feedback stabilized, nonlinear reactances should be eliminated (or at least buffered from the resonator), and all nonlinearity should be vested in one circuit having odd symmetry and the lowest possible flicker noise level. The fact that the dc bias and, hence, flicker noise density in each limiting diode both increase with drive level, even though the AM-to-PM conversion coefficient decreases, suggests that there may be an optimum drive level resulting in the lowest possible level of flicker phase noise. Such an effect was reported in reference 11 which dealt with FET oscillators.

In the cases in which the amplification and limiting functions are separable, it is possible to use reactive decoupling to reject amplifier-generated flicker noise before it enters the nonlinear block and is upconverted. This is obviously beneficial if the flicker noise power spectral density of the amplifier exceeds that associated with the nonlinear elements. For example, a properly sized coupling capacitor located between the output of a linear amplifier and the input of a limiter of the type discussed earlier would prevent the passage of amplifier-generated flicker noise into the limiter. Only a resonator would then be needed to complete the feedback loop. Of course, some minor phase correction would be needed to compensate for the carrier frequency phase shift introduced by the coupling capacitor since it is

extremely important in preventing reactive loading of the limiter caused by "off-resonance" operation.

Conclusions

From an examination of the role of amplitude-to-phase conversion in the generation of oscillator flicker phase noise, the following conclusions are presented:

1. Synthesis of odd symmetry in a feedback oscillator by combining nonlinear elements that individually lack symmetry does not prevent frequency translation of flicker noise associated with the individual nonlinear elements up to the oscillation frequency.

2. Nonlinear reactance, which is present to some extent in all semiconductor devices, is often the predominant mechanism for the transfer of flicker noise into the phase of an oscillator signal.

3. Flicker envelope (amplitude modulation) noise can be converted into flicker phase noise by a number of mechanisms, including the amplitude-to-phase conversion effect associated with reactively loaded limiters. This effect was analyzed and shown to be a significant consideration, particularly in low-frequency oscillators in which it is a dominant effect.

4. Good circuit design was shown to be an important factor in minimizing flicker phase noise.

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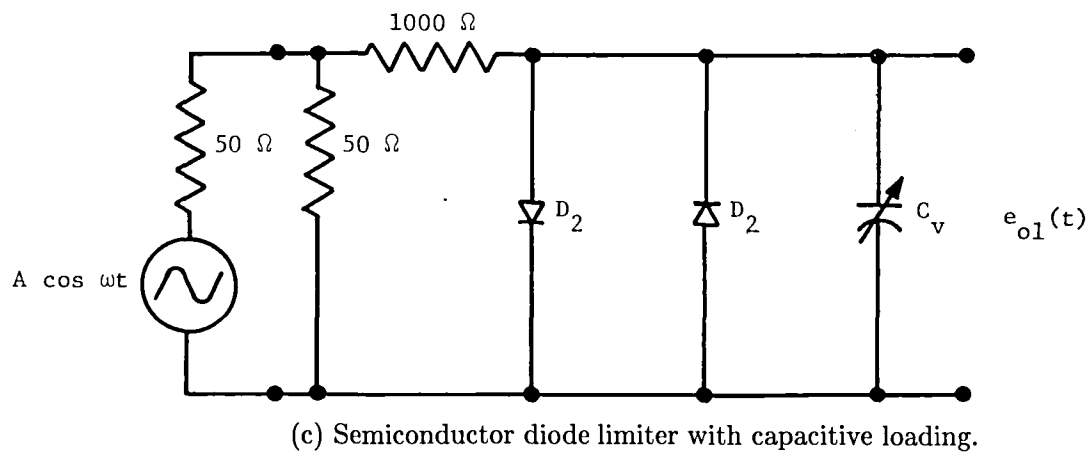
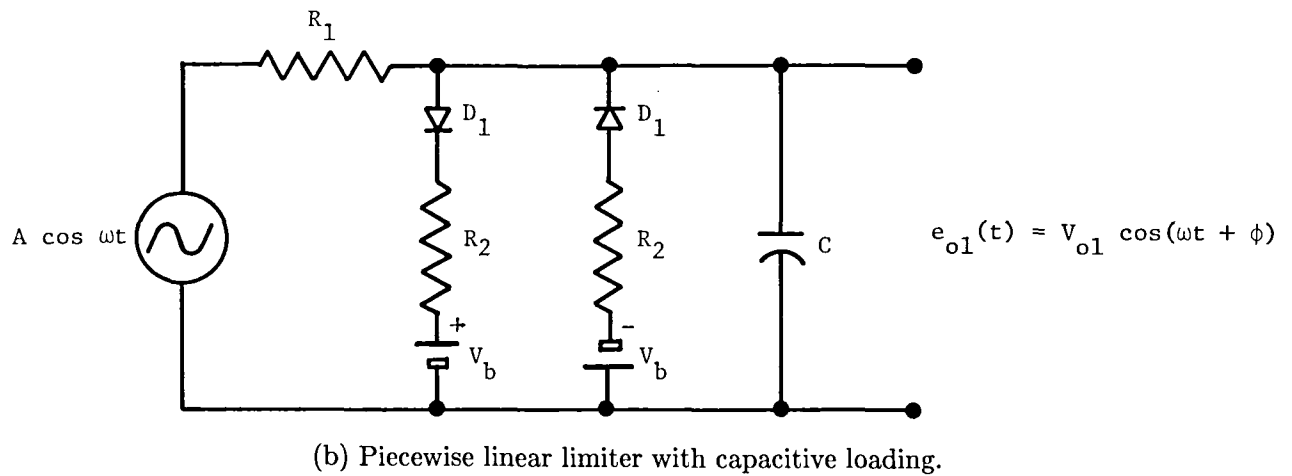
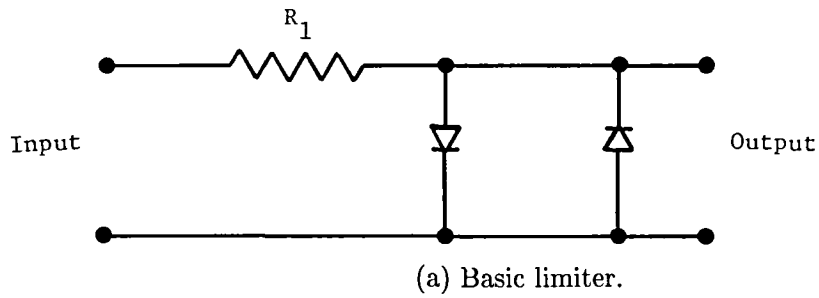


Figure 1. Limiter circuit models used.

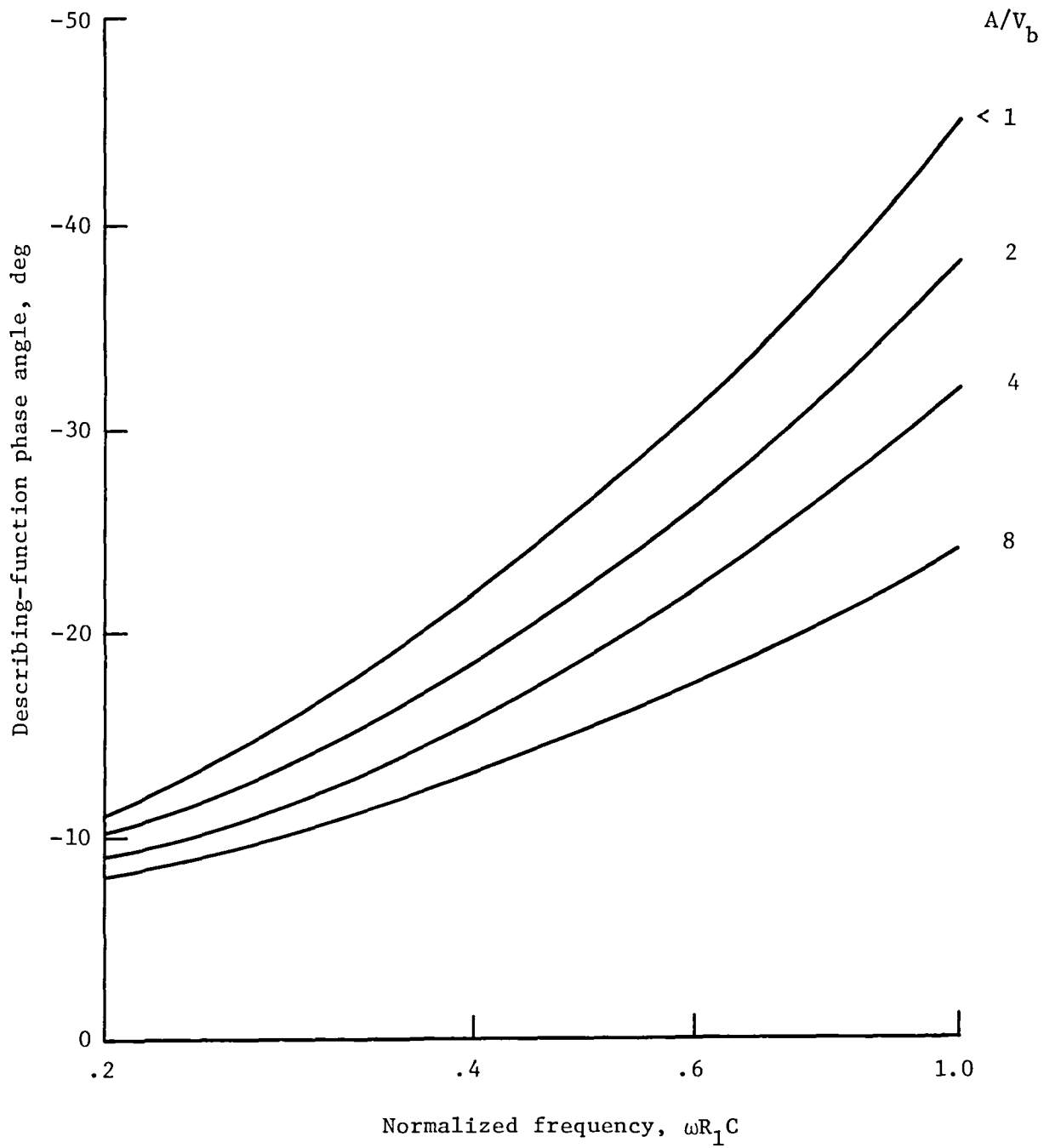


Figure 2. Describing-function phase angle plotted against excitation frequency for piecewise linear limiter circuit.

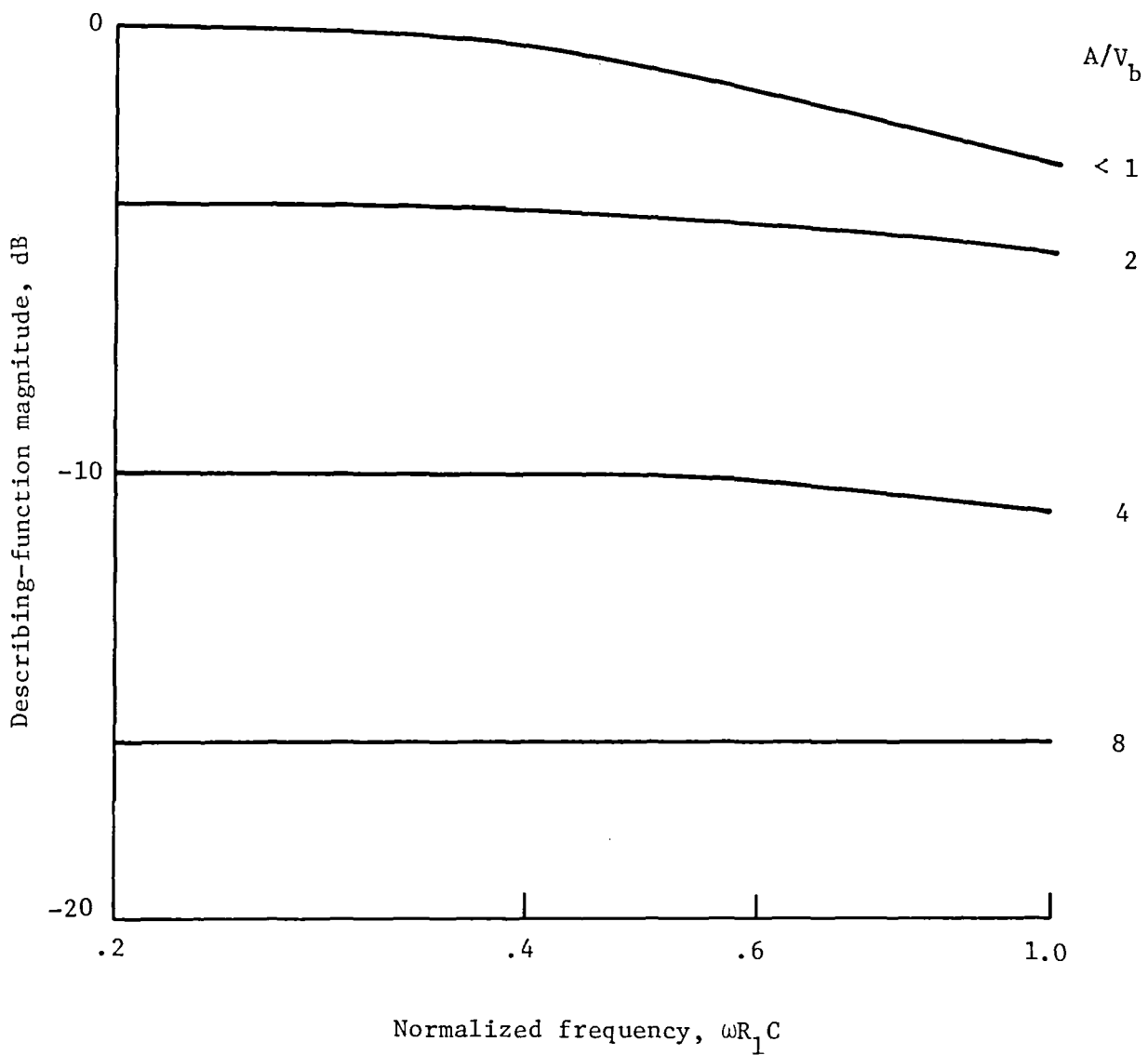


Figure 3. Describing-function magnitude plotted against excitation frequency for piecewise linear limiter circuit.

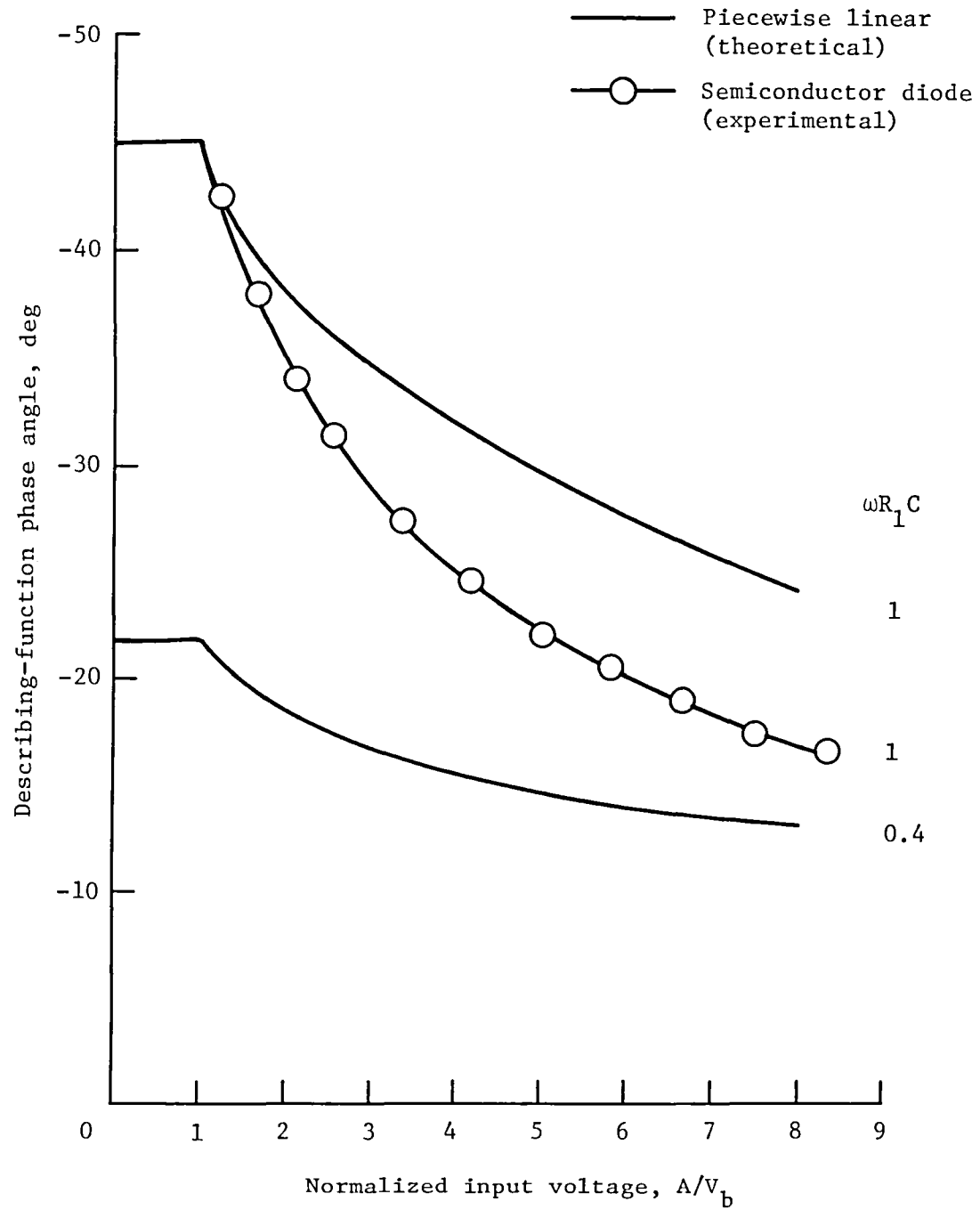


Figure 4. Theoretical and experimental phase shift plotted against input voltage for piecewise linear and semiconductor diode limiter circuits.

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